Artificial Bee Colony (ABC) based Variable Density Sampling Scheme for CS-MRI

Akshay Kumar Jagadish†, Soumya Goswami*, Pramit Saha*, Satrajit Chakrabarty* and Kasi Rajgopal‡
† Electrical and Electronics Engineering, NITK, Surathkal. *Electrical Engineering, Jadavpur University, Kolkata.
‡Electrical Engineering, Indian Institute Of Science, Bangalore.

***Abstract*—The self-sustained dynamics of the bee population in nature is a result of their hierarchical working culture, efficient organizing skills and unique highly developed foraging ability, which enables them to interact effectively among each other as well as with their environment. In this paper, a novel algorithm utilizing the bee's swarm intelligence, and its heuristics based on quality and quantity of food sources (nectars) is proposed to generate a variable density sampling (VDS) scheme for compressive sampling (CS) based fast MRI data acquisition. The algorithm uses the scout-bees for global random selection process which is further fine-tuned by employed and onlooker-bees who forage locally in the neighborhood giving prime importance to points possessing high fitness values (or high energy) usually located around the center of *k*-space. The algorithm introduces the concept of searching for the high quality food sources in annular regions, called as bins, of varying widths. Retrospective CS-MRI simulations show that the proposed *k*-ABC based VDS scheme performs significantly better than other sampling schemes.**

***Index Terms*—Artificial Bee Colony (ABC), k-ABC, Compressive sampling (CS), Variable Density Sampling (VDS).**

I. Introduction

Compressed Sensing (CS) [1], a sparse signal recovery technique, is being widely used to reconstruct MR images because unlike traditional Nyquist sampling method, it requires only few measurements provided the underlying image has sparse representation in some transform domain (like Fourier domain), and the acquired samples are incoherent in the same domain. Thereby, showing tremendous potential in reducing the data acquisition time.

With the application of compressed sensing to rapid and high quality MRI, Lustig et al. [2] developed the idea of randomized Cartesian sub-sampling. The random sampling concept was extended to draw samples from a polynomial based probability density function (pdf) using Monte-Carlo algorithm with minimum peak interference. However, constructing a pdf that will generate a suitable sampling scheme with appropriate number of low and high frequency k-space samples is a challenging problem. Ravishankar et al. [3] have proposed an adaptive sampling algorithm using a novel sparsifying Dictionary Learning technique which simultaneously learns an image patch based dictionary and reconstructs the image using under-sampled data iteratively.

Traveling Salesman Problem (TSP) based optimization for sampling scheme generation has been proposed by Chauffert et al. [4]. In this method, a continuous k-space trajectory is traced using sample points drawn from a specific target distribution (π-distribution) by solving TSP. Recently, Adcock et. al [5] have proposed the usage of optimal sub-sampling strategy based on the structure of the signal, instead of uniform random sub-sampling which yields poor results.

Artificial bee colony (ABC) algorithm is a swarm intelligence algorithm, motivated by the foraging behavior of bee swarms. Karaboga et al. [6] developed an idea based on honey bee swarm for numerical optimization and later proposed a combinatorial ABC algorithm for solving traveling salesman problem.

In this paper, we have designed a meta-heuristic iterative artificial bee colony algorithm to mimic a VDS scheme for fast MR imaging. It is shown that the proposed novel k-ABC based sampling scheme gives significant improvement over other well-known algorithms for drawing samples from the k-space.

II. k-ABC Algorithm

Motivation:

In nature, the nectar collected by the bees are stored in cells of the hive. Similarly, in MR imaging, sampling points or samples that are closer to center of k-space hold the essential features pertaining to overall structure of the image, contrast details and signal to noise ratio. Hence, certain region in the center of k-space, possessing high energy [7], is sampled densely which is analogous to the bee-hive. The concept of variation in the quantity and quality of food sources gradually with distance from the hive has been used to design the k-space sampling scheme. In order to preserve randomness in sampling scheme necessary for compressed sensing based reconstruction, random vectors or agents called scout-bees have been utilized for foraging for best nectars globally. Employed and onlooker-bees are made to forage locally in the neighborhood so as to further fine-tune the components possessing higher fitness values that are usually concentrated close to the center of k-space. Thus, a meta-heuristic algorithm has been developed given the better performance of heuristics [4] as follows:

A. Random Search by the Scout Bees

The hive of the bee system is the center or source of bee-movement. In this work, the beehive is initialized at the center of k-space representing the high energy low frequency peak of the k-space. A specified number of scout-bees perform random search and convey the information of location of food sources to unemployed-bees.

The scout-bees forage for N_k nectars from k^{th} annular region known as bin, whose width can remain either constant or increase based on its distance from the hive. The radius r_k which determines the annular region width is given by:

$$r_k = \begin{cases} r_{k-1} + c_k * \Delta r, & \text{if } k > 0 \\ r_{IN}, & \text{if } k = 0 \end{cases} \quad (1)$$

where Δr gives the incremental width of the annular region and c_k determines the relative bin width of successive bins. The initial radius r_{IN} and Δr are user-specified parameters.

The location of food sources in k^{th} bin is given by:

$$x_i^k = (rand(0,1)(r_k - r_{k-1}) + r_{k-1},\ \theta), \forall k \in \{1, 2, ...\} \quad (2)$$

where $i \in \{1, 2, 3, ...N_k\}$, $\theta \in [0, 2\pi]$ is a randomly chosen angle and $x_i^0 = (rand(0,1)r_{IN},\ \theta)$ gives the location of food sources within r_{IN}.

The fitness value of the food source at the location x_i^k is given by $f(x_i^k)$. This value indicates the probability of the food source being picked. Its value can be chosen from a standard probability distribution function like the Gaussian or any user-defined distribution which closely approximates the *k*-space distribution of the slice being imaged. The entire food source selection mechanism of *k*-ABC is performed using this pdf as underlying distribution. The entire *k*-space is spanned bin-wise, and information regarding the quality and distance of the selected food sources is passed on to the employed and onlooker-bees in the hive.

B. Local Search By the Employed-Bees

A neighborhood search is performed by employed-bees once they are exhorted to take up the *M* optimally fit locations in each bin. The bees were designed to perform this search in form of local concentric circles of varying radii r_l until at least one better position is found. If the bee fail to update its position in one cycle, they restart the process by searching in circles of larger radii, $r_{l+1} = r_l + l * \Delta r_l$. One cycle is said to be completed after nectars at J locations, y_j^l with r_l have been inspected. If a high quality nectar is found by the bee in its path, it will repeat the same cycle with the updated position as its new center. Thus, the employed-bees not only exploit the food source in its memory but also explore for other nectars available in its neighborhood. This process is carried out by all employed-bees simultaneously (or in parallel) using a greedy selection mechanism. The location of the nectars inspected by employed-bees, in the neighborhood of each of the *M* nectars, is given by:

$$y_j^l = (r_l,\ \theta + j\Delta\theta),\ j \in \{1, 2, ..J\},\ l \in \{1, 2, 3....\} \quad (3)$$
$$\text{subject to the constraint } r_l \leq r_s$$

where y_j^l indicates the locations along the circumference of circle with radius r_l, $\Delta\theta = (2\pi/J)$ and r_s is maximum radial spread for search.

Further, a greedy nectar selection mechanism was designed as follows:

$$x_{i+N_k}^k := y_j^l\ ,\ i \in \{1, 2, 3, ..., \} \quad (4)$$
$$\text{subject to the constraint } \nabla f(y_j^l) \geq \nabla f(y_{j-1}^l)$$

The equation (4) implies that the nectars are chosen based on the gradient of the fitness function at those locations. These newly chosen profitable nectars are added to the N_k food sources already present in the each bin, based on its location in *k*-space. Thus, the effective number of points in each of the bin after updating is $N_k^e \geq N_k$.

Path and time constraints are imposed on the employed-bees by restricting them within distance r_s and controlling the number of nectars being searched and their relative distances through Δ, θ and Δr_l. The parameter J determines the number of food sources searched within radius r_l.

C. Advance of the Onlookers-Bees

A designated number of the onlookers move towards the *E* food sources, once the employed-bees inform them about these nectars. The *E* best sites which serve as the base-locations for onlooker-bees are chosen from the nectars obtained in (4), if they exceed a threshold, f_{th} defined as follows:

$$x_e^k := x_i^k,\ \ \texttt{if}\ \ f(x_i^k) > f_{th}\ \ \forall\ i \in \{1, 2, ..., N_k^e\} \quad (5)$$

where $e \in \{1, 2, ..., E\}$ and *E* was fixed to be 50% of the employed-bee swarm size.

Suppose *S* onlooker-bees move to each of these *E* positions by mutual agreement. Since, only one onlooker-bee can reach the best position x_e^k at a time, the remaining *S*-1 bees will strive to occupy that position, but can only be in close proximity of x_e^k at the locations $x_{e'}^k$ such that:

$$|x_e^k \text{ - } x_{e'}^k| < \epsilon \text{ and } f(x_{e'}^k) \leq f(x_e^k).$$

Since, there is a high probability of finding a better or an equally good food source in the neighborhood of a high fitness nectar. The advance of the onlooker-bees is said to be completed when $S - 1$ bees satisfying the above constraints occupy their respective positions. The net count of the number of bees in each bin after the advance of the onlookers is N_k^o.

III. Results

We have conducted simulations to demonstrate the performance of the proposed *k*-ABC based VDS scheme for retrospective CS-MRI, using Dictionary Learning technique (DLMRI) [3] for reconstruction. Results were obtained by setting the total number of dictionary learning iterations = 10, number of *k*-SVD iteration = 20, image patch size = 36 with sparsity T_0 = 5 and simulated noise level = 0.005. Two images obtained from fully sampled k-space of in-vivo MR scan of brain and T2-Weighted Sagittal Spinal image (Fig. 1), both of dimensions 512×512 were used as reference images in our simulations. The image quality was evaluated using PSNR (Peak Signal to Noise Ratio) - higher the value better is the reconstruction, HFEN (High Frequency Normalized Error) [3] - lower the value the better and SSIM (Structural Similarity Index Measure) [8] - better when closer to one, were used as metrics for structural fidelity.

The proposed *k*-ABC VDS scheme is designed using only five parameters. Two parameters - r_{IN} and Δr have been held constant throughout for comparison, while the other parameters were chosen by Monte-Carlo simulations such that

Algorithm: k-ABC

Input : Initial Food sources N_0, Initial search radius r_{IN}, Swarm size of the employed-bees M, Bin Width Δr, Maximal radius for neighborhood search r_s and Fitness distribution $f(x)$.

Output: Variable Density Sampling scheme

1 Initialize the bee population with N_0 - food sources within the radius r_{IN}.
2 Pick N_k food sources randomly from each bin using (2).
3 Pass the information regarding the M best nectars within r_s of k-space to the employed-bees and evaluate them.
4 The employed-bees perform a neighborhood search to find better nectars locally according to (3).
5 Evaluate these nectars and select them based on the constraint in (4) which results in N_k^e samples/bin.
6 The onlooker-bees choose the E best nectars, from the available N_k^e nectars, based on a distance criterion and settle in their vicinity.
7 Memorize the locations of all food sources selected by scout, employed and onlooker-bees in the bin, N_k^o.
8 Repeat the steps **2-7** for all bins.
9 The final sampling pattern is obtained by logical OR of N_k^o over k bins.

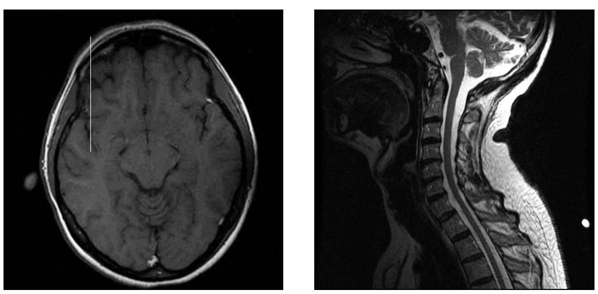

Fig. 1. **Reference Images:** In-Vivo MRI Brain Image and T2 Sagittal Spine Image

the desired under-sampling percentages are obtained. The k-space has been mapped from [$-k_{max}$, k_{max}] to [-1,1] for convenience. The parameters r_{IN} and Δr were set to 0.078 and 0.039 respectively in the above scale. The variable c_k in (1) was chosen to be unity for all values of k, resulting in a total of 12 bins of uniform width excluding the initial bin. Gaussian distribution with μ = 0 and σ = 0.39 (Fig. 2 top-left) was chosen as the underlying fitness value distribution.

The number of food sources picked by scout-bees per bin N_k is chosen to exponentially vary with distance and is given by $N_k = N_0 * exp(-z * r_k)$ where initial food source quantity N_0 is user-specified and z is a constant whose magnitude determines the rate of decay of N_k and sign determines if N_k is decreasing (z >0), increasing (z <0) or kept constant (z=0).

The Table I shows the image quality measures of the DLMRI reconstructed images for the three food source distributions at undersampling factors R = 5%, 10%, 15% and 20%. It is evident that the image quality metrics are very close to each other when N_k is distributed uniformly or exponentially decreasing. However, the results are not encouraging for N_k exponentially increasing with distance which can be attributed

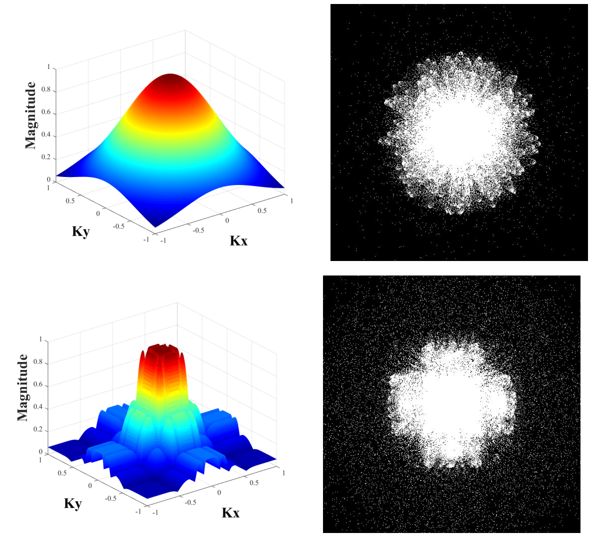

Fig. 2. **Fitness-Value Distribution Template and its Corresponding VDS Schemes:** *Left Column* : Fitness Templates- Gaussian and π-distribution, *Right Column* : Corresponding Sampling patterns obtained from k-ABC (R=20%).

TABLE I
IMAGE QUALITY MEASURE COMPARISON FOR SPINAL IMAGE FOR DIFFERENT METHODS OF FOOD SOURCE DISTRIBUTION

Food Source Distribution per bin	Quality Measure	Undersampling (R) in %			
		5	10	15	20
Exponentially Decreasing	PSNR	**31.59**	**35.88**	**38.93**	**41.21**
	SSIM	**0.8848**	**0.9458**	**0.9660**	**0.9799**
Uniform distribution	PSNR	30.80	35.04	37.85	40.67
	SSIM	0.8728	0.9256	0.9556	**0.9756**
Exponentially Increasing	PSNR	25.62	33.06	35.53	36.56
	SSIM	0.7205	0.9152	0.9406	**0.9766**

to the trade-off involved in giving high priority to bins located further away from k-space center. Therefore, the motivation for the bees to search for high quality nectars (synonymous with high energy k-space center) in the vicinity of the hive is justified by the exponentially decreasing food source distribution with distance and is chosen for all further simulations and analysis.

The ability of the proposed k-ABC sampling scheme to adapt to the underlying fitness distribution is shown in Fig. 2. The Gaussian distribution captures the typical exponentially decaying distribution of the k-space in MRI images. It has been shown that the π distribution is optimal distribution in orthogonal system that minimizes the upper bound of Rauhut's result (Theorem 4.2, 4.4 in [7]) such that it meets the sparsity constraint of CS sampling.

The Table II shows the image quality metrics of the DLMRI reconstructed images for under-sampling percentages R = 5%, 10%, 15% and 20%. Monte Carlo Simulations over the set of parameters results in 10-60 sampling patterns. It is seen that the mean of the image quality metric match closely with the best result and the corresponding variances are small indicating robustness of the reconstruction irrespective of the combination of the parameters which generate the sampling patterns.

We show the reconstructed brain images in Fig. 3 obtained from the k-ABC sampling patterns corresponding to the undersampling percentages R = 5% and 20% respectively. The sampling pattern for R = 5% is shown in Fig. 5 (top left) and Fig. 2 (top right) shows the pattern for R = 20%. The inset in

TABLE II

IMAGE QUALITY OF RECONSTRUCTION RESULTS FOR BRAIN IMAGE FOR DIFFERENT UNDER-SAMPLING PERCENTAGES (R)

Parameters			Quality Measures						
N_0	r_s	M	$R\%$	**PSNR**	PSNR(μ/σ^2)	**HFEN**	HFEN(μ/σ^2)	**SSIM**	SSIM(μ/σ^2)
1500	0.67	75	20	**43.71**	43.66/0.009	**0.30**	0.28/0.001	**0.97**	0.96/1.93e-06
1400	0.60	45	15	42.39	42.48/0.004	0.52	0.49/0.007	0.95	0.95/1.01e-05
200	0.50	35	10	40.86	40.85/0.013	0.80	0.80/0.002	0.94	0.93/7.57e-05
900	0.40	10	5	37.13	36.28/0.478	1.88	2.04/0.060	0.90	0.89/0.004

TABLE III

RECONSTRUCTION QUALITY OF SPINAL IMAGE FOR DIFFERENT VDS SCHEMES FOR UNDER-SAMPLING (R) OF 20% AND 5%

VDS Scheme	PSNR	HFEN	SSIM
k-ABC (Gaussian distribution)	**41.21 (31.59)**	**0.54 (3.49)**	**0.98 (0.89)**
k-ABC (π-distribution)	40.44 (31.14)	**0.64** (3.75)	**0.98 (0.88)**
Independent π-distribution [4]	36.93 (29.19)	1.72 (4.98)	0.94 (0.84)
Power Law based distribution [2]	32.64 (26.23)	3.25 (6.61)	0.89 (0.76)

the left corner show the magnified image of the patch in the rectangle. It is clearly seen that the k-ABC sampling scheme is able to reconstruct with good fidelity at low undersampling of 5%. As a sanity test, we have compared the normalized line intensity profile of the original and reconstructed images in Fig. 4 along the vertical line (shown in Fig. 1) to demonstrate its ability to capture the fine intensity structural variations in the reconstruction for even 5% under sampling.

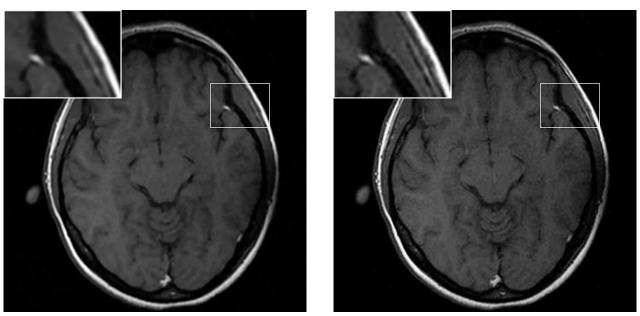

Fig. 3. **Reconstructed Images:** Reconstructed Brain Image for R=5% and 20% respectively.

Further, the image quality metrics were compared between k-ABC sampling and other VDS schemes for T2-Weighted Sagittal Spinal image for R=20% and 5% (in brackets) as shown in the Table III. Our sampling scheme (using Gaussian distribution as underlying fitness distribution) with a PSNR = 41.21(31.59), HFEN = 0.54(3.49) and SSIM = 0.97(0.88) performs significantly better than the rest of the methods. The Fig. 5 shows a reconstruction of T2-Sagittal Spine image using the k-ABC sampling scheme and the independent π distribution sampling scheme by Chauffert et al [4] at 5% undersampling.

IV. CONCLUSION

In this paper, we have proposed a novel approach to design a variable density k-space sampling scheme based on the unique foraging abilities of a bee colony. A modified artificial bee colony algorithm (k-ABC) has been developed using only few parameters, for generating a VDS sampling scheme for CS-based MR imaging. Simulation results validate that the proposed sampling scheme gives significantly better performance.

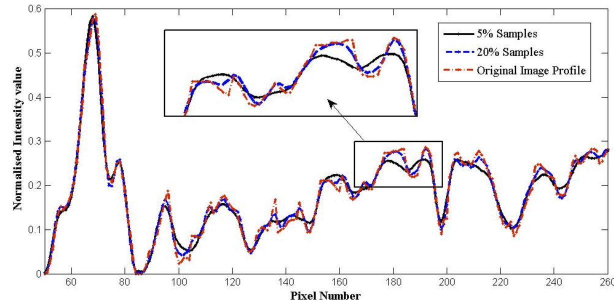

Fig. 4. **Intensity Profile Comparison :** Along the line X=160 and Y=[50:260] of In vivo Brain image as shown in Fig. 1.

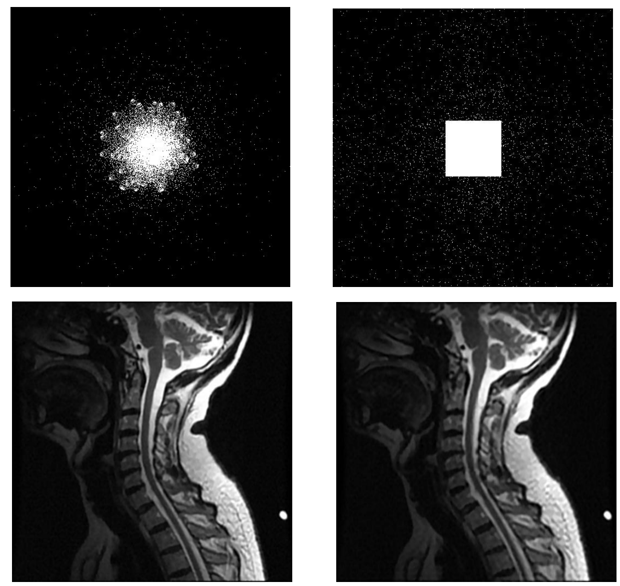

Fig. 5. **VDS Schemes and Reconstructed Images:** *Top row*: Sampling patterns for R=5% - k-ABC and Independent pi-distribution, *Bottom row* : Corresponding Reconstructed T2-Weighted Spinal image.

REFERENCES

[1] E. J. Candès, J. Romberg, and T. Tao, "Robust Uncertainty Principles: Exact Signal Reconstruction from Highly Incomplete Frequency Information," *Information Theory, IEEE Transactions on*, vol. 52, no. 2, pp. 489–509, 2006.

[2] M. Lustig, D. Donoho, and J. M. Pauly, "Sparse MRI: The Application of Compressed Sensing for Rapid MR Imaging," *Magnetic Resonance in Medicine*, vol. 58, no. 6, pp. 1182–1195, 2007.

[3] S. Ravishankar and Y. Bresler, "MR Image Reconstruction from Highly Undersampled k-space data by Dictionary Learning," *Medical Imaging, IEEE Transactions on*, vol. 30, no. 5, pp. 1028–1041, 2011. g 2011, pp. 3751–3755.

[4] N. Chauffert, P. Ciuciu, J. Kahn, and P. Weiss, "Variable Density Sampling with Continuous Sampling Trajectories," *SIAM Journal on Imaging Sciences*, 2014 vol. 7, no. 4, pp. 1962–1992, 2014.

[5] B. Adcock, A. C. Hansen, C. Poon, and B. Roman, "Breaking the Coherence Barrier: A New Theory for Compressed Sensing," *arXiv preprint arXiv:1302.0561*, 2013.

[6] D. Karaboga and B. Gorkemli, "A Combinatorial Artificial Bee Colony Algorithm for Travelling Salesman Problem," in *International Symposium on Innovations in Intelligent Systems and Applications (INISTA), 2011*, June 2011, pp. 50–53.

[7] H. Rauhut, "Compressive Sensing and Structured Random matrices," *Theoretical Foundations and Numerical Methods for Sparse Recovery*, vol. 9, pp. 1–92, 2010.

[8] Z. Wang, A. C. Bovik, H. R. Sheikh, and E. P. Simoncelli, "Image Quality Assessment: from Error Visibility to Structural Similarity," *Image Processing, IEEE Transactions on*, vol. 13, no. 4, pp. 600–612, 2004.